Exploreelearning®

Technical Research Report

EFFICACY ANALYSIS OF FRAX FOUNDATIONS I

QUASI-EXPERIMENTAL RESULTS OF FRAX USAGE AND I-READY DIAGNOSTIC MATH ASSESSMENT SCORES

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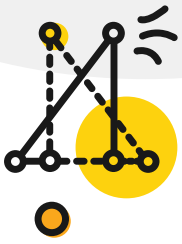
Table of Contents



EXECUTIVE SUMMARY

The current study analyzed the impact of Frax Foundations I program on the math achievement of 3rd and 4th graders attending a large school district in Florida over the course of the 2021-2022 school year. The study used a quasi-experimental methodology to isolate the impact of Frax on student achievement, by statistically matching students who completed Frax lessons throughout the school year with similar students who did not use Frax. **Overall, usage of the Frax Foundations I program by 3rd and 4th grade students resulted in statistically significant gains, including for academically at-risk students.**

Specifically, some of key findings included:

- 3rd grade students saw significant gains in mathematics achievement from Fall 2021 to Spring 2022 **with an average use of only 9.6 hours of Frax usage**.
- Fourth graders who used Frax with fidelity grew 34% more than their matched peers with no Frax usage; **this is the equivalent of 8.4 additional weeks of instruction (28% of the entire instructional year) relative to non-users.**
- Frax was found to be **3x more effective** than the average educational intervention for 3rd graders and **5x more effective** than the average educational intervention for 4th graders.
- Both 3rd and 4th graders who used Frax met growth benchmarks at higher rates than non-users.
  - Frax users in 4th grade met individual growth projections at 1.6x the rate of paired non-users and Frax users in 3rd grade met individual growth projections at 1.4x the rate of paired non-users.
  - Frax users in 4th grade averaged 111% of their typical projected growth, exceeding paired non-users by over 27 percentage points.
- Frax helped more students meet grade level proficiency expectations and growth.
  - Both 3rd and 4th graders who used Frax with fidelity were significantly more likely to reach grade level proficiency in the spring compared to matched non-users.
  - **Academically at-risk students who used Frax were 2x more likely to reach grade level proficiency in the spring compared to matched non-users.**
  - Even the highest achieving students were 20% more likely to meet or exceed grade level proficiency in the Frax usage group compared to matched non-users.

Overall, the current study found that ExploreLearning Frax Foundations 1 usage with fidelity led to (1) higher test scores, (2) larger than expected learning gains, and (3) increased upward mobility for those students most at-risk. **This evidence demonstrates that Frax Foundations 1 is an effective and efficient intervention that can be used with all levels of learners and can support post-pandemic math recovery.**

This efficacy study was determined to meet the Every Student Success Act (ESSA) Tier 2 (Moderate) level of evidence by an independent, third-party review.

INTRODUCTION

ExploreLearning is a recognized leader in the educational software market, creating high-quality solutions for the most challenging problems in K-12 math and science learning. Our four digital programs (Reflex®, Frax®, Science4Us®, and Gizmos®) are currently used in classrooms in every state in the U.S. and more than 80 countries worldwide (1). Our programs are state- and national-standards aligned, including Next Generation Science Standards (NGSS) and the Standards for Mathematical Practice (SMP). With a philosophy of life-long learning driving our thought leadership, a careful attention to the current needs of educators in today's rapidly shifting educational culture, and a legacy of proven results, ExploreLearning is the best combination of proven expertise and innovative solutions over time to meet today's and tomorrow's educational challenges.

Performance with fractions has been a weak point in U.S. education for decades and has not improved in recent years (Siegler, 2017). Building a strong foundation of early fractions knowledge is critical to later mathematical success. In a recent national survey of 1,000 Algebra teachers, most rated students' knowledge of fractions as "poor" and rated fractions as one of the top two barriers to students mastering algebra (Hoffer et al., 2007). Additionally, fractions knowledge in grade five uniquely predicts students' mathematics achievement in high school. This is true even after controlling for other variables like general intellectual ability, working memory, and family income and education levels (Siegler et al., 2012), making interventions to support early fractions learning an important and effective way to support later academic achievement.

ExploreLearning Frax is a standards-aligned program designed to support fractions learning for students in grades 3-5 using research-proven instructional methods. Game-based and story-driven, Frax invites students to travel through space on engaging and standards-based missions that motivate and incentivize student-driven learning. Students earn rewards and tokens as they play, which they can use to personalize their virtual living quarters on the ship. It uses innovative adaptive technology that delivers different levels of support to different students depending on their progress, making it effective for both struggling students and those needing extra practice via a learning path that is unique to their skills and abilities. Frax also provides real-time data to show teachers when a student is struggling so that they can intervene.

The current report details the findings from an efficacy analysis of Frax Foundations I as a digital complement to the existing math curriculum in a large, suburban public school district in Florida. Foundations I, broadly aligned to grade 3 fractions standards, is centered on the conceptual understanding that fractions are numbers with magnitude just like any other number and builds a

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1) For additional information on all the ExploreLearning products, please visit https://explorelearning.com/

foundation for learning fractions arithmetic (1). Foundations I is designed as a zero-entry program, so that students with no previous knowledge of fractions can begin using the program immediately. The goal of the study was to measure the impact of Frax on student achievement through a quasi-experimental method that supports causal inferences. The current report includes a total of 2,530 3rd and 4th grade students who used Frax Foundations I during the 2021-2022 school year, and a matched sample of 2,530 3rd and 4th grade students who did not use Frax (2). The dependent variable was change in students' scale scores on the i-Ready Diagnostic math assessment from fall to spring. Broad math scale scores were used on the current analyses, providing the most stringent test of the efficacy of our product.

This study meets Every Student Succeeds Act (ESSA) Tier 2 (Moderate) levels of evidence according to the U.S. Department of Education guidelines. The study uses quasi-experimental matching methods to create baseline equivalency between treatment and control groups along major confounding factors and criteria for statistically significant positive effects.

METHODOLOGY

Case Control Matching to Create Equivalent Groups

While true randomized experiments (i.e. students each randomly assigned to either the control or the experimental condition) are often seen as the "gold standard" in research for determining causal relationships between interventions and outcomes, randomization in a school setting can often be difficult. Here, we use a quasi-experimental design to account for pre-existing differences between groups and support causal inferences between the intervention (Frax usage) and the outcome (change in i-Ready Diagnostic math scale scores from fall to spring). The statistical package SPSS was used to create case-control matches of a student with Frax usage (treatment) to a very similar student with no Frax usage (control). Students were matched on current grade level (3rd or 4th), fall i-Ready Diagnostic scores (within 5 scale score points), and current and prior Reflex program usage (within 5 days usage) (3).

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1) Frax Foundations II, aligned to 4th grade standards, was released in March 2023. No students in the current sample had access to Foundations II at the time of data collection.

2) While Frax Foundations I is geared towards meeting 3rd grade standards, 4th grade students can benefit from a review of previously learned content to help students solidify foundational fractions concepts and prepare them for more advanced fractions addition and subtraction in 4th grade standards. Analyses in this report will discuss 3rd and 4th grade findings separately to account for these likely differences in students' preparation levels and prior exposure to classroom fractions instructions.

3) The district is also a user of another ExploreLearning math program, Reflex. Reflex is a game-based program that supports math fact fluency. In order to isolate the effects of the Frax program in the current study, we matched students on number of days usage of Reflex in the prior and current school years. Thus, we can be confident that the current report shows the impact of Frax usage above and beyond any impact from Reflex. Additional reports on Reflex efficacy can be found at https://explorelearning.com/

Independent Variable: Frax Foundations I

Frax is an adaptive, game-based, online program for students in grades 3–5 to help all students learn fractions. Frax Foundations I is designed as a zero-entry program so that students with no previous knowledge of fractions can begin using Frax immediately. Frax automatically recognizes and delivers different levels of support to different students depending on what they need to progress. As students navigate Frax's game-based activities, and based on their specific interactions, the program adapts to what they demonstrate they do not know and reinforces skills as needed for students to begin making sense of fractions concepts. There is also hinting functionality for students who are struggling with a particular question. These features create individualized, efficient instruction that intentionally moves each student forward on a customized path by continuously adapting and scaffolding instruction based on how they interact with the program.

The Frax approach teaches students that fractions are numbers. Students learn that fractions have a magnitude (size) and expand the number system beyond whole numbers in useful ways. Students practice these concepts in Frax through explicit, scaffolded use of length models, number lines, and measurements that help broaden their understanding of numbers, or number sense, to include fractions. By building conceptual understanding of fraction magnitude, students build the foundation they need to move beyond memorized procedures and tricks and instead learn to make sense of fractions arithmetic.

Frax is continually assessing and monitoring students' progress formatively rather than using separate summative assessment activities. Thus, progress through the successive missions is the best demonstration of student growth, and the metric used in the current study. Frax missions typically require 20–30 minutes to complete. Missions must be completed in order and students are limited to completing one mission per day. Because missions are adaptive, the questions in each mission will be tailored to students' performance which supports and maintains student engagement and motivation.

Dependent Variable: i-Ready Diagnostic

i-Ready Diagnostic assessment scores were used to measure student's mathematics achievement. In all of the analyses here, overall math scale scores were used to provide the most stringent test of the impact of Frax on student's overall ability to perform grade-level mathematics. As detailed above, fall scale scores were used to create matched pairs in the control and treatment groups.

At both fall and spring, i-Ready classifies students into one of three criterion-referenced grade-level placements based on scale scores: on grade level or above, 1 grade level below, and 2 or more grade levels below. The on grade level placement is further divided into three range levels: early, mid, and late. By the end of the school year (spring testing), students who fall into the mid-grade level or above categories are considered to meet or exceed grad level proficiency, and those in the early-on grade level

category are approaching grade level proficiency. Additionally, typical growth is calculated as the average expected annual growth based on their placement level at baseline. This provides a metric for assessing whether that student's academic growth over the course of the school year was above or below reasonable expectations based on typical academic instruction. Weekly expectations of growth can be calculated as 1/30th of a student's typical growth score.

In the current study, academic growth was measured by (1) the absolute change in math scale scores between the fall and spring assessment administrations (2) percentage of typical expected growth achieved (absolute change in scale scores divided by typical growth) and (3) changes in relative placement level.

Study Sample

The students come from a large, suburban public school district in the state of Florida. The district has over 97,000 students across 118 schools. The district's minority enrollment is over 60% and 35% of students are economically disadvantaged. The current sample included all students in 3rd and 4th grade with both fall and spring i-Ready assessment scores and valid data on Frax and Reflex program usage. Due to relatively low sample sizes within each group, MLL students and students with current 504 plans were excluded from the current analyses. The case control matching procedure resulted in a treatment group of 2,530 3rd and 4th grade students who used Frax Foundations I, and a matched control group of 2,530 3rd and 4th grade students who did not use Frax. Mean i-Ready Diagnostic fall (pretest) math scale scores between treatment and control groups differed by .04 points and satisfies WWC standards for baseline equivalence (< .05).

Table 1: Student sample characteristics by treatment group

	Treatment (Frax use)	Control (No Frax use)
Total N's	2,530	2,530
Fall 2021 Math Scale Score: M (SD)	439.68 (24.05)	439.64 (24.02)
Starting placement levels: n (%)		
2+ grade levels below	509 (20%)	516 (20%)
1 grade levels below	1,499 (59%)	1,469 (58%)
Early-on grade and above	522 (21%)	545 (22%)
Grades		
3	1,220	1,220
4	1,310	1,310
Demographics: n (%)		
Male	1,219 (48%)	1,180 (47%)
Female	1,256 (50%)	1,293 (51%)
Hispanic/Latino	866 (34%)	887 (35%)
Black/African American	413 (16%)	364 (14%)

Frax program usage varied widely across the sample. The Frax Foundations I program contains 27 missions, and student progress is quantified here as the number of missions completed. To understand the impact of dosage, quartiles were calculated in the full sample and preliminary analyses conducted across completion rate bands. No significant differences were found between the first and second quartiles for either grade, so those are combined into one "low completion" band. The High Completion group (>75% completion) represents students that have used the program with fidelity. See Table 2 for the details within each band.

Table 2: Student usage details for each grade level and Frax usage band

		N	Missions completed range	Missions completed M (SD)	Total hours spent in missions: M (SD)
3rd grade students	Low Frax usage	679	1-10	4.86 (2.88)	2.82 (1.95)
	Moderate Frax usage	286	11-19	14.24 (2.34)	7.45 (2.05)
	High (Fidelity) Frax usage	255	20-27	25.66 (2.20)	12.05 (3.61)
4th grade students	Low Frax usage	817	1-10	4.51 (2.80)	2.33 (1.66)
	Moderate Frax usage	244	11-19	14.32 (2.62)	6.72 (1.88)
	High (Fidelity) Frax usage	249	20-27	25.51 (2.39)	11.49 (3.92)

RESULTS

Frax Usage and Scale Score Increases

The first analysis looked at absolute gain in scale scores from fall to spring. Paired samples t-tests found that fidelity Frax users (completing 20+ missions) gained significantly more points between fall and spring i-Ready assessments compared to matched users with no Frax usage (Table 3). This was true for both 3rd and 4th grade students. For 3rd graders, we also saw significant gains for users relative to matched non-users in the moderate completion category (11-19 missions completed).

Table 3: Gain in i-Ready Diagnostic math scale scores from fall to spring for matched groups of users and non-users in 3rd and 4th grade by usage category (low, moderate, high)

	3rd grade		
	Treatment	Control	Difference
Low usage (1-10)	21.48	22.86	-1.38
Moderate usage (11-19)	24.77	21.21	3.56***
High usage (20+ missions)	25.74	21.71	4.03***
	4th grade		
	Treatment	Control	Difference
Low usage (1-10)	20.61	20.38	.23
Moderate usage (11-19)	22.41	21.38	1.03
High usage (20+ missions)	25.20	18.87	6.33***
*p<.05 **p<.01 ***p<.001			

Standardized effect sizes (Hedge's g) were used to determine the magnitude of changes in diagnostic gain. The effect size for third graders who used Frax with fidelity (high completion rates) was 0.23 and for fourth graders it was .37. Previous research has found an average effect size of .08 for elementary school-level educational interventions measured by standardized test measures (Lipsey et al., 2012). This means that Frax was 3x more effective than the average educational intervention for 3rd graders and 5x more effective than the average intervention for 4th graders.

Using the annual typical growth measure for students who used Frax with fidelity (high completion rates), the benefit of Frax translates to an additional 4.7 weeks of instruction (or 16% of the entire school year) relative to non-users for 3rd graders and 8.4 weeks of instruction (or 28% of the entire school year) relative to non-users for 4th graders.

Frax Usage and Progress towards Typical Growth

The next analyses looked at students' attainment of typical or expected growth. On average, students with no or low Frax usage (0-10 missions completed) failed to meet 100% typical growth projections (Table 4). Paired samples t-tests found that 3rd grade students with moderate to high Frax completion rates and 4th grade students with high Frax completion achieved a higher percentage of expected growth than their matched peers with no Frax usage. For instance, 4th grade students with fidelity Frax usage averaged 111% of their typical projected growth, exceeding paired non-users by over 27 percentage points.

Table 4: Paired-samples t-test results of mean % of expected typical growth by users and non-users

	3rd grade		
	Treatment	**Control**	**Difference**
Low usage (1-10)	81.6%	86.8%	-5.2%
Moderate usage (11-19)	94.7%	81.1%	+13.6%***
High usage (20+ missions)	100.2%	84.49%	+15.71%***
	4th grade		
	Treatment	**Control**	**Difference**
Low usage (1-10)	90.0%	88.9%	+1.1%
Moderate usage (11-19)	97.9%	93.2%	+4.7%
High usage (20+ missions)	111.0%	83.2%	+27.8%***
**p<.05 **p<.01 ***p<.001*			

Categorical 2x2 chi-square tests were also used to test for differences in the amount of students who fully met their typical growth expectation score compared to those who did not fully meet their typical growth expectation score across the Frax completion rate groups and matched control groups (Table 5). Frax users in both 3rd and 4th grades met individual growth projections at significantly higher rates than non-users. For instance, 4th graders with fidelity Frax usage were on average 1.6x more likely to meet their growth projections than the group of matched non-users, while 3rd graders with high or moderate Frax usage were 1.4x more likely to meet growth projections as compared to matched non-users.

Table 5: Chi-square results of students who met typical growth expectations by users and non-users

	3rd grade		
	Treatment	**Control**	**Chi-square**
Low usage (1-10)	33.1%	35.2%	0.64
Moderate usage (11-19)	43.7%	30.4%	10.82***
High usage (20+ missions)	44.7%	32.5%	7.95**
	4th grade		
	Treatment	**Control**	**Chi-square**
Low usage (1-10)	39.5%	40.9%	0.31
Moderate usage (11-19)	43.4%	41.0%	0.31
High usage (20+ missions)	55.4%	34.5%	21.94***
**p<.05 **p<.01 ***p<.001*			

Frax Usage and Meeting Grade Level Proficiency

The final set of analyses looked at the potential of Frax to support students in approaching grade level proficiency, as determined by i-Ready Diagnostics relative placement categories. Categorical 2x2 chi-square tests found that both 3rd and 4th grade high Frax users were more likely to approach or meet grade level proficiency expectations in the spring, compared to matched non-users. Table 6 shows the increase in the number of students per group who were in the approaching or meeting grade level proficiency categories in the spring compared to fall (presented as increase in percentage points) and the associated chi-square results.

Table 6: Change in percentage points of 3rd and 4th grade fidelity users and non-users in the On-Grade level category (Approaching, Meeting, or Exceeding Proficiency) from Fall to Spring

	3rd grade		
	Treatment	**Control**	**Chi-square**
Low usage (1-10)	+34%	+37.7%	2.00
Moderate usage (11-19)	+40.6%	+34.3%	2.42
High usage (20+ missions)	+49.8%	+40.4%	4.56*
	4th grade		
	Treatment	**Control**	**Chi-square**
Low usage (1-10)	+38.7%	+37.4%	.26
Moderate usage (11-19)	+44.7%	+38.3%	.39
High usage (20+ missions)	+45.8%	+37.4%	3.65*
***p<.05 **p<.01. ***p<.001*			

Figure 1 shows the mobility across i-Ready Placement levels from fall to spring for students with no Frax usage compared to 3rd and 4th grade students with high Frax completion. For reporting of spring results, the on grade level range has been separated out into 'approaches proficiency' (early on grade level) and 'meets or exceeds proficiency' (mid or above grade level) to reflect definitions of i-Ready Diagnostic relative grade level placements and associated proficiency standards. Within each fall student placement category (2+ grade levels below, 1 grade level below, on grade level or above), Frax usage was related to increased mobility across the school year. 86% of high Frax users who placed 2 or more grade levels below in the fall moved up one or more grade levels by the end of the school year, compared to only 72% of matched non-users. **These academically at-risk Frax users were also 2x more likely to approach or meet grade level proficiency and half as likely to remain in the 2+ grade levels below category compared to matched non-users.** 78% of high Frax users who placed one grade level below in the fall reached the on-grade level range (approaching or meeting/exceeding proficiency) by the end of the school year, compared to only 63% of matched non-users. Frax was also beneficial for maintaining the achievement status of students who were already on-track for meeting grade level proficiency in the fall; **non-users were 3.5x more likely to fall short of fully meeting or exceeding grade level proficiency by the end of the year compared to matched Frax users.** 94% of the high Frax users who were on-track for meeting grade level proficiency in the fall fully met or exceeded grade level expectations in the spring, compared to only 78% of matched users in the control group.

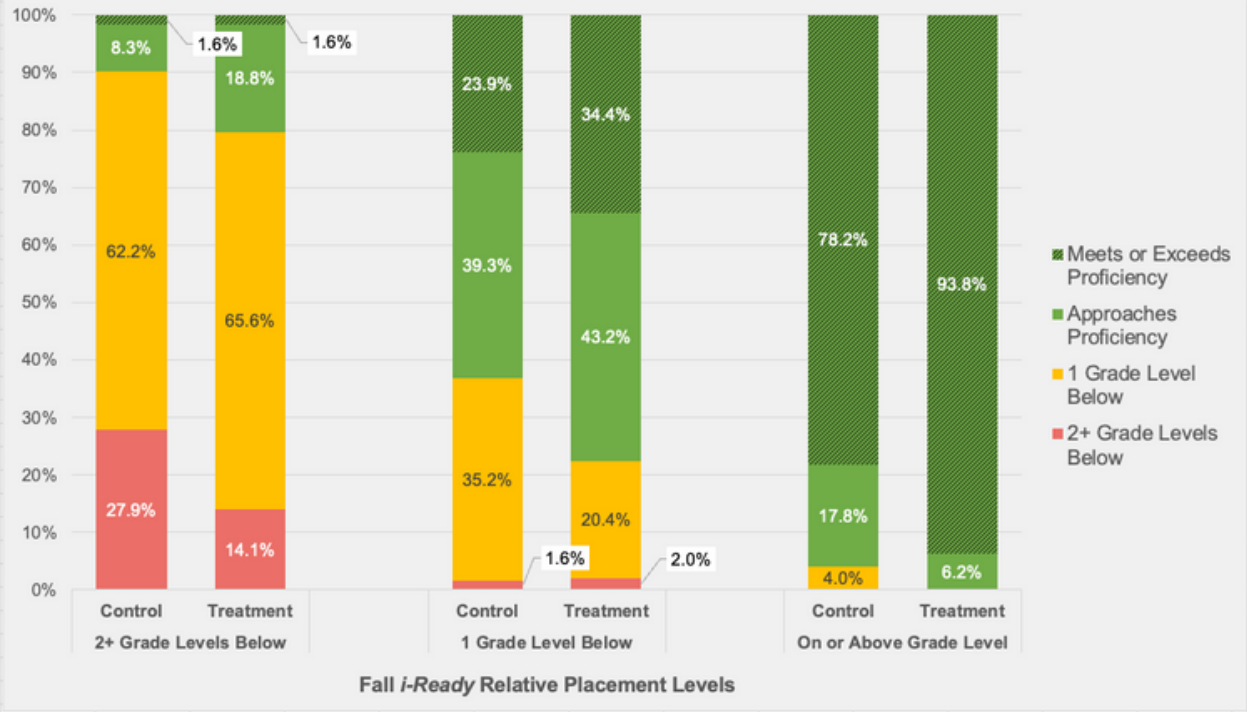

Figure 1: *Spring i-Ready relative placement levels based on fall i-Ready relative placement levels for control students (0 missions) vs high usage treatment students (20+ missions).*

CONCLUSIONS

The current analysis provides evidence of the impact of the Frax Foundations I program on student math achievement. Performance with fractions has been a weak point in U.S. education for decades (Siegler, 2017) and building a strong foundation of early fractions knowledge is critical to later mathematical success (Hoffer et al., 2007). The current study found that both 3rd and 4th graders benefited from the support provided by the Frax program. These students were better equipped with a foundational understanding of fractions concepts, compared to matched peer non-users, and we observed increased achievement in terms of individual growth and demonstration of grade-level proficiency.

Among our sample of over 5,000 case-control matched 3rd and 4th grade students, we found that fidelity program usage led to greater academic growth between fall and spring administrations of the i-Ready Diagnostic math test. For 3rd grade students, spending an average of 9.6 hours using Frax resulted in improvements equivalent to an additional 5 weeks of instructional time. For fourth graders, an average of 11.4 hours using Frax resulted in improvements equivalent to over 8 weeks of instruction relative to matched peer non-users. Standardized effect sizes revealed that Frax was 3x more effective than the average educational intervention for 3rd graders and 5x more effective than the average educational intervention for 4th graders. Schools often have a number of existing interventions and supplemental curricular programs to fit into limited time, and need to make evidence-based decisions over the adoption of additional technology. The results presented here provide compelling evidence that Frax can provide large benefits within relatively small blocks of time.

Frax also helped students make the progress needed to achieve grade level proficiency expectations. Frax users in both 3rd and 4th grade were more likely to meet or exceed their individual projected growth scores compared to paired non-users. This shows evidence that Frax supports meaningful academic growth above and beyond the growth that would be expected of these students in a typical educational setting without any intervention.

The current study also found evidence that Frax helps academically at-risk students catch up to their peers. Frax users who placed 2 or more grade levels below in the fall were 2x more likely to approach or meet grade level proficiency and half as likely to remain in the 2+ grade levels below category compared to matched non-users. There is also a strong benefit for students on track to meet grade level proficiency at baseline; non-users in this group were 3.5x more likely to fall short of fully meeting grade level proficiency by the end of the year compared to matched Frax users.

A recent longitudinal report from Cambium Assessment found significant learning loss among students in math, with the number of students observed on grade level mathematics dropping as much as 21 percentage points. Reports such as this one have led to widespread concerns over helping students recover from learning loss. The current study found that fidelity usage of Frax Foundations I, which can be reached by the average student in 11-12 hours, led to higher diagnostic scores, larger than expected learning gains, and upward mobility for those students most at-risk, strongly supporting its use as a math recovery tool for 3rd and 4th graders.

ABOUT EXPLORELEARNING

ExploreLearning LLC, based in Charlottesville, VA, was founded in 1999 by educators looking for new ways to inspire students across grades K–12 and help them succeed in math and science. With a philosophy of life-long learning driving our thought leadership, a careful attention to the current needs of educators in today's rapidly-shifting educational culture, and a legacy of proven results, ExploreLearning is the best combination of proven expertise and innovative solutions over time to meet today's and tomorrow's educational challenges.

Our four digital programs (Reflex®, Frax®, Science4Us®, and Gizmos®) are currently used in classrooms in every state in the U.S. and more than 80 countries worldwide. Our programs are state- and national-standards aligned, including Next Generation Science Standards (NGSS) and the Standards for Mathematical Practice (SMP). ExploreLearning is a recognized leader in the educational software market, earning many major edtech awards.

We aim to foster student success through the use of galvanizing, age-appropriate multimedia, including interactive simulations, STEM case studies, adaptive games, instructional videos, and much more. Our development team of engineers, researchers, and instructional-design experts, most of whom are former educators, are continually innovating beyond the latest advancements in instructional pedagogy and edtech. Our programs support students in developing mastery of fundamental skills and deep conceptual understanding in math and science, while also fully engaging them in the process of internalized learning, promoting growth mindset, resiliency, productive struggle, and perseverance.

Our goal is to provide educators with captivating, best-in-class digital learning in math and science that helps students reach their full potential. We firmly believe that teachers are mission-critical, i.e., the greatest influence on student success. We also believe that data, instruction, and practice, when operating in tandem, are paramount to improving student learning and academic achievement. In support of these foundational beliefs, we deliver curricula, professional learning, and implementation and technical support services that:

- Combine research-proven instructional methods and innovative technology
- Enable equitable access to math and science learning for all students
- Build strong, lasting foundations for student success by developing procedural and conceptual understanding
- Supplement core curricula with flexible digital and blended implementation
- Create positive outcomes and results for both students and teachers

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